Self-gravitating Line Sources of Weak Hypercharge

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We explore the role of the Cremmer-Scherk mechanism in the context of low energy effective string theory by coupling the antisymmetric 3-form gauge field to an Abelian gauge potential carrying weak hypercharge. The theory admits a class of exact self-gravitating solutions in the spontaneously broken phase in which dual fields acquire massive perturbative modes. Despite the massive nature of these fields they admit non-perturbative progressive longitudinal modes that together with pp-type gravitational waves travel in a direction of a line source at the speed of light.

Considerable effort has been devoted to the search for classical string-like solutions in relativistic field theories. Such solutions range from the pioneering work on vortices as models for dual strings [1] to more recent investigations on the properties of global and superconducting cosmic strings [2]-[4]. A common feature in recent work has been the role played by singular sources as models for the strings themselves. Such source descriptions often lend themselves to a formulation in terms of de Rham periods. Thus in the Higgs vacuum of the global Abelian Higgs model [3], the phase θ of a complex scalar field satisfies the massless field equations $d * d\theta = 0$ in a regular space-time domain. In such a domain one may introduce a 2-form potential $\hat{B}$ by $d\hat{B} = *d\theta$. Classical sources enter the theory as solutions with $\int_C d\theta = 2\pi$ for some closed space-like curve $C = \partial\Sigma_2$ bounding a space-like disc Σ_2. For such solutions, $\hat{B}$ can be promoted to a distribution on space-time satisfying $d * d\hat{B} = 2\pi\delta$ with $\int_{\Sigma_2} \delta = 1$. One then identifies the solution as a line source threading C at each instant and such "axionic" strings have interesting cosmological implications [5].

In a recent note we have suggested that fields arising in low energy effective string actions may have consequences for the standard model of the electroweak interactions [7]. By coupling the antisymmetric 3-form gauge field H to an Abelian gauge potential 1-form A carrying weak hypercharge via a gauge covariant derivative of the standard Higgs weak isospinor, we showed explicitly how the masses of the $W^{\pm}, Z^0$ could depend on this coupling. A salient feature of this generalised Cremmer-Scherk mechanism [6] was the manner in which the H field became assimilated into the physical degrees of freedom of the vector bosons via a spontaneous breakdown of a local gauge symmetry. Since the H field along with the dilaton ϕ is thought to have implications for cosmology, it is of interest to explore the gravitational sector of the low energy effective string action in the presence of the Cremmer-Scherk interaction. Although the model discussed in Ref.[7] involves the full non-Abelian $SU(2) \times U(1)$ gauge theory of the electroweak standard model, we shall here restrict attention to a single local Abelian internal symmetry gauge group for simplicity but retain the hypercharge interpretation. The Cremmer-Scherk mechanism is controlled by a coupling constant λ and we are interested in the phase with $\lambda \neq 0$. Thus to lowest order in string fields we investigate the dynamics derived from the action density 4-form

$$\Lambda[\mathbf{g}, \phi, A, B] = \kappa\mathcal{R} * 1 - \frac{(2\alpha - 3)}{4} d\phi \wedge *d\phi + \frac{1}{2} e^{-2\phi} dB \wedge *dB + \frac{1}{2} e^{-2\phi} dA \wedge *dA + \lambda A \wedge dB \tag{1}$$

where A is a 1-form, B a 2-form, ϕ the dilaton 0-form on spacetime M with a metric $\mathbf{g} = \eta_{ab} e^a \otimes e^b$, curvature scalar $\mathcal{R}$ and associated Hodge map $*$. The field equations derived from (1) by varying $A, B, \phi, \mathbf{g}$, respectively, are

$$d(e^{-2\phi} * dA) + \lambda dB = 0, \tag{2}$$

$$d(e^{-2\phi} * dB) - \lambda dA = 0, \tag{3}$$

$$d * d\phi = \frac{2}{(2\alpha - 3)} e^{-2\phi} (dB \wedge *dB + dA \wedge *dA), \tag{4}$$

$$\frac{\kappa}{2} R_{bc} \wedge *(e_a \wedge e^b \wedge e^c) = \tau_a[\phi] + \tau_a[A] + \tau_a[B], \tag{5}$$

where

$$\tau_a[\phi] = \frac{(2\alpha-3)}{4}(\iota_a d\phi * d\phi + d\phi\iota_a * d\phi)$$
$$\tau_a[A] = \frac{1}{2}e^{-2\phi}(\iota_a dA \wedge *dA - dA \wedge \iota_a * dA)$$
$$\tau_a[B] = \frac{1}{2}e^{-2\phi}(\iota_a dB \wedge *dB + dB \wedge \iota_a * dB) \tag{6}$$

in terms of the interior operator with $\iota_a(e^b) = \delta^b_a$. In a regular source-free domain of space-time (2) and (3) imply

$$d\tilde{A} = \lambda e^{2\phi} * \tilde{B}, \tag{7}$$

$$d\tilde{B} = \lambda e^{2\phi} * \tilde{A}, \tag{8}$$

in terms of the variables $\tilde{A} = A - \frac{1}{\lambda}df_0$, $\tilde{B} = B - \frac{1}{\lambda}df_1$ in the gauge equivalence classes $[A]$ and $[B]$, respectively. One may fix gauges by taking solutions with particular f_0 and f_1. Using (7) or (8) the entire theory can be recast in terms of either the fields $\{\mathbf{g}, \phi, \tilde{A}\}$ or the fields $\{\mathbf{g}, \phi, \tilde{B}\}$, and the two descriptions refer to dual sectors of the same theory. Moreover, in terms of the $\{\mathbf{g}, \phi, \tilde{A}\}$ description the theory admits vector fields satisfying a generalised Einstein-dilaton-Proca system:

$$d(e^{-2\phi} * d\tilde{A}) + \lambda^2 e^{2\phi} * \tilde{A} = 0, \tag{9}$$

$$d * d\phi = -\frac{2\lambda^2}{(2\alpha-3)}e^{2\phi}\tilde{A} \wedge *\tilde{A} + \frac{2}{(2\alpha-3)}e^{-2\phi}d\tilde{A} \wedge *d\tilde{A}, \tag{10}$$

$$\frac{\kappa}{2}R_{bc} \wedge *(e_a \wedge e^b \wedge e^c) = \tau_a[\phi] + \tau_a[\tilde{A}] + \frac{\lambda^2}{2}e^{2\phi}(\iota_a * \tilde{A} \wedge \tilde{A} + *\tilde{A} \wedge \iota_a\tilde{A}). \tag{11}$$

It is clear how in the absence of gravitation the vector field $\tilde{A}$ acquires massive propagating modes. In a similar manner the theory admits a description in terms of $\{\mathbf{g}, \phi, \tilde{B}\}$ satisfying the generalised Einstein-dilaton-massive-Kalb-Ramond system:

$$d(e^{-2\phi} * d\tilde{B}) - \lambda^2 e^{2\phi} * \tilde{B} = 0, \tag{12}$$

$$d * d\phi = -\frac{2\lambda^2}{(2\alpha-3)}e^{2\phi}\tilde{B} \wedge *\tilde{B} + \frac{2}{(2\alpha-3)}e^{-2\phi}d\tilde{B} \wedge *d\tilde{B}. \tag{13}$$

$$\frac{\kappa}{2}R_{bc} \wedge *(e_a \wedge e^b \wedge e^c) = \tau_a[\phi] + \tau_a[\tilde{B}] + \frac{\lambda^2}{2}e^{2\phi}(*\tilde{B} \wedge \iota_a\tilde{B} - \iota_a * \tilde{B} \wedge \tilde{B}). \tag{14}$$

Working with the fields $\{\mathbf{g}, \phi, \tilde{A}\}$, and restricting to cylindrical symmetry we seek solutions for $\tilde{A}$ and ϕ with the metric

$$\mathbf{g} = du \otimes dv + dv \otimes du + d\rho \otimes d\rho + \rho^2 d\psi \otimes d\psi + 2\mathcal{H}(u,\rho)du \otimes du, \tag{15}$$

in a coordinate system (u, v, ρ, ψ). We take $\mathcal{H}$ to have the form

$$\mathcal{H} = f(u)^2 h(\rho) \tag{16}$$

and

$$\tilde{A} = f(u)\beta(\rho)du \tag{17}$$

with the dilaton constant,

$$\phi = \phi_0. \tag{18}$$

It follows from (7) that the corresponding solution for $\tilde{B}$ will have the form

$$\tilde{B} = -\frac{e^{-2\phi_0}}{\lambda} f(u)\beta'(\rho)\, du \wedge \rho d\psi. \tag{19}$$

The equations (9), (10) and (11) are satisfied provided the functions $\beta(\rho)$ and $h(\rho)$ solve

$$\beta'' + \frac{1}{\rho}\beta' - \mu_0^2\beta = 0, \tag{20}$$

$$e^{2\phi_0}\kappa(h'' + \frac{1}{\rho}h') + (\beta')^2 + \mu_0^2\beta^2 = 0 \tag{21}$$

where $\mu_0 = \lambda e^{2\phi_0}$. We seek smooth solutions to these equations for $\rho > 0$ such that $d\tilde{A}$ tends to zero as $\rho \to \infty$ and the gravitational field tends to that of a cylinder with arbitrary gravitational mass (which may be zero). We recall that $\mathcal{H}(u,\rho) = 2\pi\sigma_0\, ln\rho$ yields a weak field Newtonian limit corresponding to a cylinder of mass density σ_0 per unit length. Therefore we require that $h(\rho) \sim C\, ln\rho$ as $\rho \to \infty$. Such solutions exist for arbitrary $f(u)$ in terms of modified Bessel functions:

$$\beta(\rho) = K_0(\mu_0\rho), \tag{22}$$

$$\begin{aligned} \frac{\kappa}{{\mu_0}^2} e^{2\phi_0}\, h(\rho) = &\int_1^\rho \rho' ln\rho' (K_1(\mu_0\rho')^2 + K_0(\mu_0\rho')^2) d\rho' \\ &+ \frac{1}{2}\rho^2 ln\rho\, (K_0(\mu_0\rho)K_2(\mu_0\rho) - (K_0(\mu_0\rho))^2) + C\, ln\rho. \end{aligned} \tag{23}$$

where C is an arbitrary non-negative constant. The profiles $\beta(\rho)$, $h(\rho) - {\mu_0}^2 \frac{C}{\kappa} e^{-2\phi_0}\, ln\rho$ are displayed in Figure 1 for $\kappa = \mu_0 = 1, \phi_0 = 0$.

The interpretation of these solutions depends on the form of $f(u)$. When f is constant the solution is static. In terms of the coordinates (t, x, y, z) where

$$t = \frac{1}{\sqrt{2}}(u+v) \quad , \quad z = \frac{1}{\sqrt{2}}(v-u) \quad , \quad x = \rho\cos\psi \quad , \quad y = \rho\sin\psi$$

we identify $\rho = 0$ as a line source for $\{\mathbf{g}, \tilde{A}\}$ along the z-axis at each instant. Writing the field strength $d\tilde{A}$ in terms of hyper-electric e and hyper-magnetic b fields with respect to dt, one finds that a radial e emanates from this source and it is everywhere transverse to b. The fact that $\int_{C_1} b$ for a closed space-like contour C_1 and the flux of e through a finite space-like cylinder depend on the extent of the integration regions is a reflection of the massive nature of the $\tilde{A}$ field.

When $f(u)$ is a non-constant bounded function, the solution describes a progressive gravitational wave with amplitude $h(\rho)$ that propagates together with $\tilde{A}$ with amplitude $\beta(\rho)$ in the z direction at the speed of light. In the other spatial directions the $\tilde{A}$ field falls off exponentially to zero at infinity while the behaviour of the gravitatonal field is determined by $h(\rho)$. If one interprets the singular domain $\rho = 0$ as a straight wire, then it acts as a kind of gravitational optical fibre that guides a pp-type gravitational wave. Such an interpretation has potential astrophysical implications.

Given the surprising properties of these exterior self-gravitating Einstein-Proca solutions it may be of interest to explore the generalised Cremmer-Scherk mechanism [7] on the gravitational sector of low energy effective string theory. It may be of further interest to note that any Einstein-Proca solution can be used to generate a solution to non-Riemannian theories of gravity [8].

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